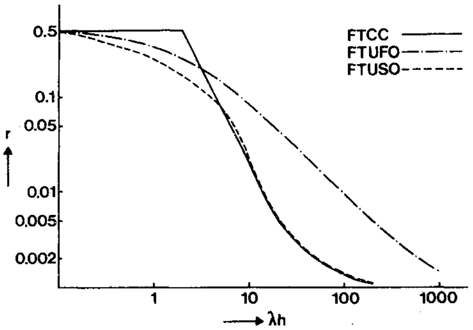

Fig. 1 Stability ranges for three difference schemes.

wind first-order space (FTUFO) schemes:

FTCC

$$r \le \begin{cases} \tfrac{1}{2} & \text{if } \lambda h \le 2 \\ 2/\lambda^2 h^2 & \text{if } \lambda h > 2 \end{cases} \tag{19}$$

FTUFO

$$r \le 1/(2+\lambda h) \quad \text{for all } \lambda h \tag{20}$$

It is clear from Fig. 1 that the stability range of FTUSO is worse than those of FTCC and FTUFO for all values of λ and h. Consequently, we must use much smaller values of the time step Δt with FTUSO in order to obtain a stable computational procedure.

Conclusions

We solved Eq. (2) numerically using the three finite-difference schemes for various values of λ, h, and different boundary conditions. For small values of λh ($\lambda h \le 2$), the central difference scheme CC is recommended. Even when USO gives the same accuracy as CC, USO is computationally more complex. For $\lambda h > 2$, the use of USO at $x = x_1, \ldots, x_{N-2}$, together with UFO at $x = x_{N-1}$ as recommended by Atias et al.,[1] gave about the same order of accuracy as the use of UFO at all points. This result may seem to be unexpected because the truncation error of the USO is $O(h^2)$. However, boundary conditions play an important role in the determination of the overall discretization error.

For time-dependent problems, CC has the best stability range and is the most accurate scheme for $\lambda h < 2$. For $\lambda h > 2$, both CC and USO have very stringent stability requirements as compared to the UFO. If implicit time-differencing is used, then FTUSO could be stabilized in the same way that FTCC has been stabilized by the use of alternating direction implicit methods by Atias et al.[1] However, whether USO would yield a more accurate solution is an open question. The extensions of these results to two dimensions and to the nonlinear Navier-Stokes equations are under investigation and will be reported in a subsequent paper.

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Integral Method Improvement for Computation of Transonic Shock Wave—Turbulent Boundary-Layer Interactions

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Introduction

IN this Note, we present some improvements of the Lees and Reeves-Klineberg integral method[1] extended to the shock wave-turbulent boundary-layer interactions in supersonic and transonic flowfields. This method requires considerably less computing time than the Navier-Stokes equations solution of the two-dimensional transonic problem[2] and seems suitable for the three-dimensional interaction calculation. The integral method for turbulent boundary-layer interaction was developed in hypersonic cases[3] but using rough turbulence modeling, which give worse results than in the laminar interaction computations.

In a previous study,[4] we performed the interaction calculations using similar velocity profiles with an equilibrium two-layer model for the eddy viscosity. In the present work, the profiles are computed with a relaxation procedure between their equilibrium and frozen values. On the other hand, the influence is shown of the different viscous-inviscid flow couplings on the supercritical behavior of the turbulent boundary layer and the associated wall pressure distribution. Finally, this method is applied to a transonic interaction at $M = 1.4$ and comparisons with our experimental results are given for the wall pressure distribution.

Analysis

Solution of the Differential Equations

The integral equation of continuity, momentum, and first velocity moment are,[3] respectively, for adiabatic flow and $\partial p/\partial y = 0$

$$\left(H + \frac{1+m_e}{m_e}\right)\frac{\mathrm{d}\delta_i^*}{\mathrm{d}X} + \delta_i^*\frac{\mathrm{d}H}{\mathrm{d}X} + f\,\frac{\delta_i^*}{M_e}\,\frac{\mathrm{d}M_e}{\mathrm{d}X} = \frac{\tan\theta}{m_e} \tag{1}$$

$$H\frac{\mathrm{d}\delta_i^*}{\mathrm{d}X} + \delta_i^*\frac{\mathrm{d}H}{\mathrm{d}X} + (2H+1)\,\frac{\delta_i^*}{M_e}\,\frac{\mathrm{d}M_e}{\mathrm{d}X} = 0 \tag{2}$$

$$J\frac{\mathrm{d}\delta_i^*}{\mathrm{d}X} + \delta_i^*\frac{\mathrm{d}J}{\mathrm{d}X} + \frac{3J\delta_i^*}{M_e}\,\frac{\mathrm{d}M_e}{\mathrm{d}X} = D \tag{3}$$

where

$$\mathrm{d}Y = \frac{a_e\rho}{a_\infty\rho_\infty}\mathrm{d}y \qquad \mathrm{d}X = \frac{a_e\rho_e}{a_\infty\rho_\infty}\mathrm{d}x$$

Following the Lees and Reeves procedure, the integral quantities are expressed as polynomials of the similar incompressible velocity profile parameters, and the external streamline inclination θ is described by the Prandtl-Meyer relation. This three-equation system is solved using a numerical Runge-Kutta technique. The initial conditions are

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taken from experimental data in order to take into account the exact Mach and Reynolds numbers and initial velocity shape. When the initial boundary layer is supercritical, the Klineberg jump equations are used. We apply his computational procedure to pass smoothly through the critical point when the final boundary layer again becomes supercritical.

Profile Representation—Eddy Viscosity Models

In most integral formulations the appropriate similar solutions are used to determine the interdependence of the different integral quantities which are represented in polynomial form.

In this work, particular attention has been given to the eddy viscosity models in the similar velocity profiles.

In the first step, an equilibrium eddy viscosity model for both separated and attached flow region is considered. Following Alber,[5] we compute the separated velocity profiles using a two-layer model:

$$\epsilon_i = 0.018 u_e y \left[1 - \exp\left(\frac{\left(-y\sqrt{\frac{\mathrm{d}p}{\mathrm{d}x}\frac{y}{\rho}}\right)}{26\nu} \right) \right]^2 \qquad \epsilon_i < \epsilon_0$$

$$\epsilon_0 = 0.0168 u_e \tilde{\delta}^* \gamma \qquad \epsilon_0 < \epsilon_i$$

where γ is the intermittency factor issued from the Fielder and Head measurements in the vicinity of separation:

$$\gamma = \frac{1}{2}\,\mathrm{erfc}\left[5.44\left(\frac{y-\delta_z}{\delta-\delta_z}\right) - .9\right]$$

(δ_2 corresponds to the $u=0$ line)

The two-layer Cebeci-Smith-Mosinskis[6] eddy viscosity model is used to compute similar solutions for the attached velocity profiles:

$$\epsilon_i = 0.16 y^2 \left[1 - \exp\left(\frac{-y}{26\nu}\sqrt{\frac{\tau_w}{\rho} + \frac{\mathrm{d}p}{\mathrm{d}x}\frac{y}{\rho}}\right)\right]^2 \left|\frac{\mathrm{d}u}{\mathrm{d}y}\right| \qquad \epsilon_i < \epsilon_0$$

$$\epsilon_0 = 0.0168 u_e \delta^* \gamma \qquad \epsilon_0 < \epsilon_i$$

With the intermittency factor γ obtained from the Fielder and Head attached profile measurements

$$\gamma = \frac{1}{2}\,\mathrm{erfc}\left[5\left(\frac{y}{\delta} - 0.78\right)\right]$$

The Alber similar solutions are computed for a Reynolds number $R_{e\delta^*}$ of 15,000. Considering the small influence of the higher Reynolds numbers (except for accelerated flows which are not considered here), we use the same value for all computations.

In the second step, the relaxation of the eddy viscosity has been considered. Indeed, as shown experimentally by Rose and numerically in the Navier-Stokes equation solution by Shang and Hankey, the equilibrium model does not seem suitable for strong interactions. The better results are obtained using a relaxation viscosity model that is intermediate between the frozen model (where ϵ is computed with the undisturbed boundary layer characteristics), and the equilibrium one (where ϵ is a function of the local properties of the boundary layer, as previously seen).

Such a relaxation model

$$\epsilon = \epsilon_{\mathrm{eq}} - (\epsilon_{\mathrm{eq}} - \epsilon_0)\exp\left(-\frac{(x-x_0)}{\lambda}\right)$$

($\lambda = 5\delta_0$ is the relaxation length)

recently used by Hung and McCormack[2] cannot be used in the integral method procedure where polynomial forms of the integral quantities are independent of the abscissa x. Therefore, we made the hypothesis that the relaxation can be extended to the polynomials. The computations show that the $J(H)$ and $Z(H)$ [which appears in the f function, Eq. (1)] polynomials are close to the frozen form, then the preceding relaxation procedure is only applied to $D(H)$:

$$D(H) = D_{\mathrm{equil}} - (D_{\mathrm{equil}} - D_{\mathrm{frozen}})\exp(-(x-x_0)/\lambda)$$

Coupling Procedure

Another possible way to improve the integral methods is to employ an appropriate coupling procedure between external flow and boundary layer. Three coupling relations are considered in this work.

Following Van Dyke, a singular perturbation analysis shows that an $R_e^{-1/2}$ expansion of both external and boundary-layer flows gives a second-order coupling condition on displacement thickness δ^* and inviscid mass flow ρu at $y=0$:

$$\left(\frac{v}{u}\right)_{y=0} = \frac{\mathrm{d}\delta^*}{\mathrm{d}x} + \frac{\delta^*}{(\rho u)_{y=0}}\frac{\mathrm{d}(\rho u)_{y=0}}{\mathrm{d}x} \tag{4}$$

The Taylor series expansion of this relation gives at $y=\delta^*$

$$(\rho v)_{y=\delta^*} = (\rho u)_{y=0}\frac{\mathrm{d}\delta^*}{\mathrm{d}x} \text{ approximated by } \left(\frac{v}{u}\right)_{y=\delta^*} = \frac{\mathrm{d}\delta^*}{\mathrm{d}x} \tag{5}$$

and the expansion at $y=\delta$ is:

$$\left(\frac{v}{u}\right)_{y=\delta} = \frac{\mathrm{d}\delta^*}{\mathrm{d}x} - \frac{(\delta-\delta^*)}{(\rho u)_{y=\delta}}\frac{\mathrm{d}(\rho u)_{y=\delta}}{\mathrm{d}x} \tag{6}$$

These three coupling procedures make f different in Eq. (1):

$$f = 3(H+1) + \frac{1}{M_e}$$

$$f = H\left(\frac{m_e}{1+m_e}\frac{\gamma+1}{\gamma-1} + 2\right) + \frac{3\gamma-1}{\gamma-1}$$

$$f = H\left(\frac{m_e}{1+m_e}\frac{\gamma+1}{\gamma-1} + 2\right) + \frac{3\gamma-1}{\gamma-1} + \frac{M_e^2-1}{m_e(1+m_e)}Z$$

for wall, δ^*, and δ coupling, Eqs. (4-6), respectively.

The wall coupling always leads to a subcritical boundary layer behavior which eliminates the jump procedure.[7] The δ^* coupling gives a supercritical flow for $M \geq 2$ and $H=0.75$. For the δ coupling, the supercritical behavior appears at $M=1.3$. Moreover, as an advantage, the two first coupling procedures eliminate the Z polynomial which depends on the boundary-layer thickness δ definition.

Results and Discussion

A typical improvement of the integral method is shown in Fig. 1 for the interaction at $M=4$ on a 25 deg ramp. Present computed pressure distribution is in better agreement with experimental data of Thomke and Roshko than the results of Hunter and Reeves. These authors used separated laminar velocity profiles with a wake form for the turbulent viscosity, whereas we apply an equilibrium model. A δ coupling is used in both cases. Difficulties appear in the upstream influence prediction: the strong interaction solution starts with a jump so that a good agreement at the corner and on the ramp is obtained to the prejudice of the interaction beginning position.

On Fig. 2, an attempt is made to summarize the effects of coupling and turbulent viscosity models in the case of a 25 deg ramp interaction at $M=3$. Experimental data are due to Law[8]

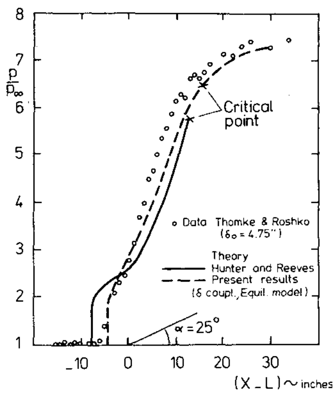

Fig. 1 Comparison of experimental and theoretical pressure distribution (ramp-$M_\infty = 4$).

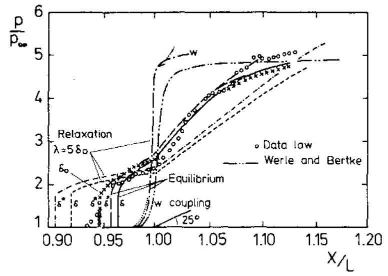

Fig. 2 Effects of coupling and eddy viscosity models (ramp-$M_\infty = 3$).

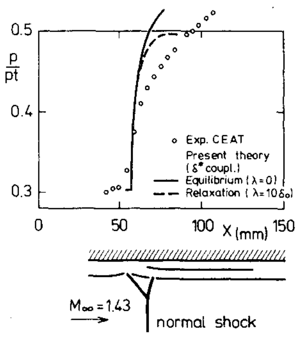

Fig. 3 Application to the transonic interaction ($M_\infty = 1.43$).

and previous Werle and Bertke[9] computations are compared with our theoretical results. The authors use a finite difference technique with δ^* coupling, equilibrium, and frozen viscosity models. A time relaxation procedure on δ^* coupling suppresses the jump. The present method, using a wall coupling, which also avoids the jump, gives similar results. The relaxation or equilibrium viscosity model does not introduce noticeable differences. The same models are used with both δ and δ^* coupling procedures. The last one seems the best, for the two turbulence representations, with an advantage for the relaxation one. Note the large influence of the relaxation length λ for the same δ^* coupling. The principal conclusion is the improved capability of the δ^* coupling/relaxation turbulence integral method to compute an interaction in good agreement with experimental results.

The first attempt to apply this method to the normal shock wave-turbulent boundary-layer interaction is shown in Fig. 3. Experimental results were obtained[10] by a normal-shock blockage ("second throat" type) of a 65×85 mm^2 test section at $M = 1.43$ and for $R_e = 10^5$/m. Static pressure is measured on the tunnel wall. The calculations are conducted only for the supersonic external flow region using a δ^* coupling. The location of the interaction is fixed by adjustment of the pressure after the jump to the experimental value. Compare the results for a large relaxation parameter ($\lambda = 10\delta_0$) with that obtained for equilibrium turbulent viscosity ($\lambda = 0$). The transonic external flow calculations are presently being continued and viscous-inviscid coupling being studied in order to give an assessment of the method for rapid computation in practical transonic cases.

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Ionospheric Doppler Sounder for Detection and Prediction of Severe Storms

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Introduction

THE correlation of atmospheric acoustic-gravity waves and severe storms has been investigated sporadically during the past twenty years. Tepper[1,2] proposed that the

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